Problem 1

Eclipses of the Jupiter's Satellite

A long time ago before scientists could measure the speed of light accurately, O Römer, a Danish astronomer studied the time eclipses of the Jupiter's satellite. He was able to determine the speed of light from observed periods of a satellite around the planet Jupiter. Figure 1 shows the orbit of the earth E around the sun S and one of the satellites M around the planet Jupiter. (He observed the time spent between two successive emergences of the satellite M from behind Jupiter).

A long series of observations of the eclipses permitted an accurate evaluation of the period of M. The observed period T depends on the relative position of the earth with respect to the frame of reference SJ as one of the coordinate axes. The average time of revolution is T_0 = 42h 28 m 16s and maximum observed period is (T_0 + 15)s.

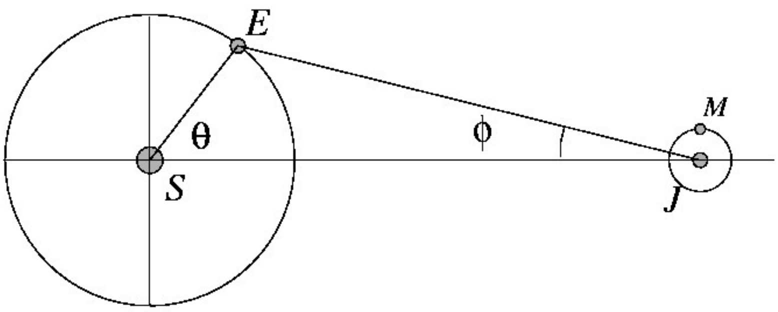

Figure 1 : The orbits of the earth E around the sun and a satellite M around Jupiter J. The average distance of the earth E to the Sun is R_E = 149.6 x 10^6 . The maximum distance is $R_{E,max}$ = 1.015 R_E. The period of revolution of the earth is 365 days and of Jupiter is 11.9 years. The distance of the satellite M to the planet Jupiter R_M= 422 x 10^3 km.

a. Use Newton's law of gravitation to estimate the distance of Jupiter to the Sun. Determine the relative angular velocity ω of the earth with respect to the frame of reference Sun-Jupiter (SJ). Calculate the speed of the earth with respect to SJ.

b. Take a new frame which Jupiter is at rest with respect to the Sun. Determine the relative angular velocity ω of the earth with respect to the frame of reference Sun-Jupiter (SJ). Calculate the speed of the earth with respect to SJ.

c. Suppose an observed saw M begin to emerge from the shadow when his position was at θ_k and the next emergence when he was at θ_{k+1} , k = 1,2,3,… From these observations he got the apparent periods of revolution T (t_k) as a function of time t_k from Figure 1 and then use an approximate expression to explain how the distance influences the observed periods of revolution of M. Estimate the relative error of your approximate distance.

d. Derive the relation between $d(t_k)$ and $T(t_k)$. Plot period $T(t_k)$ as a function of time of observation t_k. Find the positions of the earth when he observed maximum period, minimum period and true period of the satellite M.

e. Estimate the speed of light from the above result. Pont out sources of errors of your estimation and calculate the order of magnitude of the error.

f. We know that the mass of the earth = 5.98×10^{24} kg and 1 month = 27d 7h 3m. Find the mass of the planet Jupiter.