Question 3 LIGHT DEFLECTION BY A MOVING MIRROR

Reflection of light by a relativistically moving mirror is not theoretically new. Einstein discussed the possibility or worked out the process using the Lorentz transformation to get the reflection formula due to a mirror moving with a velocity $\vec{v}$. This formula, however, could also be derived by using a relatively simpler method. Consider the reflection process as shown in Fig. 3.1, where a plane mirror M moves with a velocity $\vec{v} = v\hat{e}_x$ (where $\hat{e}_x$ is a unit vector in the x-direction) observed from the lab frame F. The mirror forms an angle ϕ with respect to the velocity (note that $\phi \le 90^0$, see figure 3.1). The plane of the mirror has **n** as its normal. The light beam has an incident angle α and reflection angle β which are the angles between $\vec{n}$ and the incident beam 1 and reflection beam $1'$, respectively in the laboratory frame F. It can be shown that,

$$\sin\alpha - \sin\beta = \frac{v}{c}\sin\phi\sin(\alpha+\beta) \tag{1}$$

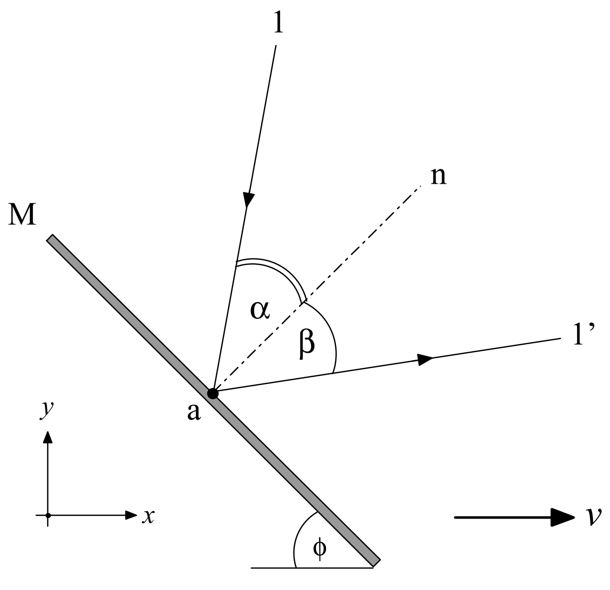

Figure 3.1. Reflection of light by a relativistically moving mirror

3A. Einstein's Mirror (2.5 points)

About a century ago Einstein derived the law of reflection of an electromagnetic wave by a mirror moving with a constant velocity $\vec{v} = -v\,\hat{e}_x$ (see Fig. 3.2). By applying the Lorentz transformation to the result obtained in the rest frame of the mirror, Einstein found that:

$$\cos\beta = \frac{\left(1+\left(\frac{v}{c}\right)^2\right)\cos\alpha - 2\frac{v}{c}}{1-2\frac{v}{c}\cos\alpha + \left(\frac{v}{c}\right)^2} \qquad (2)$$

Derive this formula using Equation (1) without Lorentz transformation!

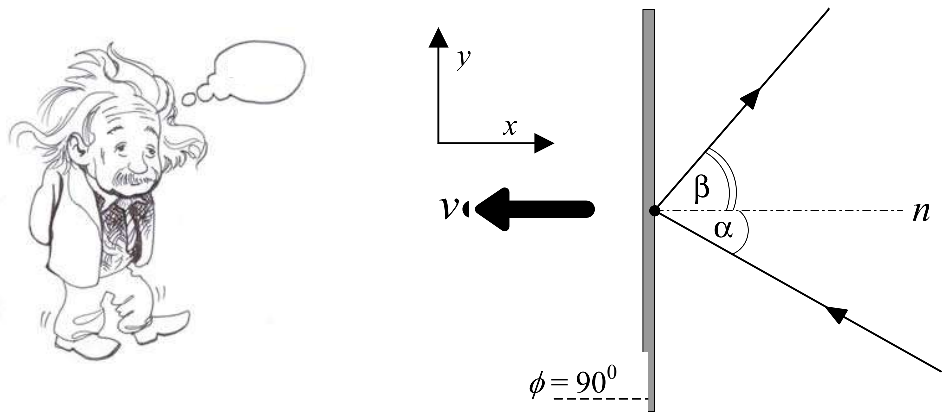

Figure 3.2. Einstein mirror moving to the left with a velocity *v*.

3B. Frequency Shift (2 points)

In the same situation as in 3A, if the incident light is a monochromatic beam hitting M with a frequency *f*, find the new frequency f' after it is reflected from the surface of the moving mirror. If $\alpha = 30^0$ and $v = 0.6\,c$ in figure 3.2, find frequency shift Δf in percentage of f.

3C. Moving Mirror Equation (5.5 Points)

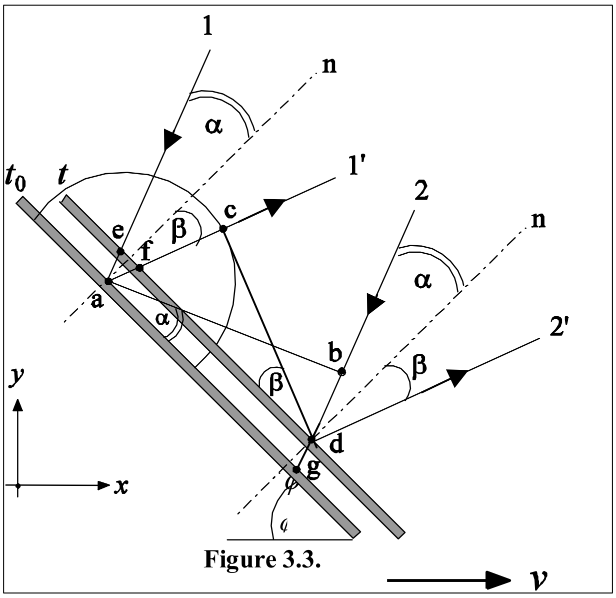

Figure 3.3.

Figure 3.3 shows the positions of the mirror at time t_0 and t. Since the observer is moving to the left, the mirror moves relatively to the right. Light beam 1 falls on point *a* at t_0 and is reflected as beam $1'$. Light beam 2 falls on point *d* at t and is reflected as beam $2'$. Therefore, $\overline{ab}$ is the wave front of the incoming light at time t_0. The atoms at point are disturbed by the incident wave front $\overline{ab}$ and begin to radiate a wavelet. The disturbance due to the wave front $\overline{ab}$ stops at time t when the wavefront strikes point *d*. The semicircle in the figure represents wave-front of the wavelet at time *t*.

By referring to figure 3.3 for light wave propagation or using other methods, derive equation (1).

Country no	Country code	Student No.	Question No.	Page No.	Total No. of pages

ANSWER FORM
3

3A) Einstein's Mirror

Proof:

3B. Shift Frequency

Frequency Shift =

3C. Moving Mirror Equation

Proof: