Theoretical Question 2

Oscillator damped by sliding friction

Theoretical Introduction

In mechanics, one often uses so called phase space, an imaginary space with the axes comprising of coordinates and moments (or velocities) of all the material points of the system. Points of the phase space are called imaging points. Every imaging point determines some state of the system.

When the mechanical system evolves, the corresponding imaging point follows a trajectory in the phase space which is called phase trajectory. One puts an arrow along the phase trajectory to show direction of the evolution. A set of all possible phase trajectories of a given mechanical system is called a phase portrait of the system. Analysis of this phase portrait allows one to unravel important qualitative properties of dynamics of the system, without solving equations of motion of the system in an explicit form. In many cases, the use of the phase space is the most appropriate method to solve problems in mechanics.

In this problem, we suggest you to use phase space in analyzing some mechanical systems with one degree of freedom, i.e., systems which are described by only one coordinate. In this case, the phase space is a two-dimensional plane. The phase trajectory is a curve on this plane given by a dependence of the momentum on the coordinate of the point, or vice versa, by a dependence of the coordinate of the point on the momentum.

As an example we present a phase trajectory of a free particle moving along x axis in positive direction (Fig.1).

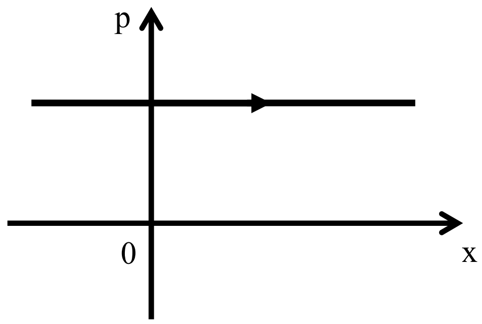

Fig. 1. Phase trajectory of a free particle.

Questions

A. Phase portraits (3.0)

A1. **[0.5 Points]** Make a draw of the phase trajectory of a free material point moving between two parallel absolutely reflective walls located at $x = - L/2$ and $x = L/2$.

A2. Investigate the phase trajectory of the harmonic oscillator, i.e., of the material point of mass m affected by Hook's force $F = - k\,x$:

a) **[0.5 Points]** Find the equation of the phase trajectory and its parameters.

b) **[0.5 Points]** Make a draw of the phase trajectory of the harmonic oscillator.

A3. **[1.5 Points]** Consider a material point of mass m on the end of weightless solid rod of length L, another end of which is fixed (strength of gravitational field is g). It is convenient to use the angle α between the rod and vertical line as a coordinate of the system. The phase plane is the plane with coordinates $(\alpha, d\alpha/dt)$. Study and make a draw of the phase portrait of this pendulum at arbitrary angle α. How many qualitatively different types of phase trajectories K does this system have? Draw at least one typical trajectory of each type. Find the conditions which determine these different types of phase trajectories. (Do not take the equilibrium points as phase trajectories). Neglect air resistance.

B. The oscillator damped by sliding friction (7.0)

When considering resistance to a motion, we usually deal with two types of friction forces. The first type is the friction force, which depends on the velocity (viscous friction), and is defined by $\boldsymbol{F} = -\gamma \boldsymbol{v}$. An example is given by a motion of a solid body in gases or liquids. The second type is the friction force, which does not depend on the magnitude of velocity. It is defined by the value $F = \mu N$ and direction opposite to the relative velocity of contacting bodies (sliding friction). An example is given by a motion of a solid body on the surface of another solid body.

As a specific example of the second type, consider a solid body on a horizontal surface at the end of a spring, another end of which is fixed. The mass of the body is m, the elasticity coefficient of the spring is k, the friction coefficient between the body and the surface is μ. Assume that the body moves along the straight line with the coordinate x ($x = 0$ corresponds to the spring which is not stretched). Assume that static and dynamical friction coefficients are the same. At initial moment the body has a position $x=A_0$ $(A_0>0)$ and zero velocity.

B1. **[1.0 Points]** Write down equation of motion of the harmonic oscillator damped by the sliding friction.

B2. **[2.0 Points]** Make a draw of the phase trajectory of this oscillator and find the equilibrium points.

B3. **[1.0 Points]** Does the body completely stop at the position where the string is not stretched? If not, determine the length of the region where the body can come to a complete stop.

B4. **[2.0 Points]** Find the decrease of the maximal deviation of the oscillator in positive x direction during one oscillation ΔA. What is the time between two consequent maximal deviations in positive direction? Find the dependence of this maximal deviation $A(t_n)$ where t_n is the time of the n-th maximal deviation in positive direction.

B5. **[1.0 Points]** Make a draw of the dependence of coordinate on time, $x(t)$, and estimate the number N of oscillations of the body?

***Note*:**

Equation of the ellipse with semi-axes a and b and centre at the origin has the following form:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$