Estimation of Unknown Resistance Using the Micro:bit's Analog-to-Digital Converter (ADC)

In physics, indirect derivation from observable variables is a standard approach for quantities that are difficult to measure directly. In this task, we will find out how to measure an unknown resistance using the micro:bit's built-in ADC (Analog-to-Digital Converter) to read voltage, even though its precision is somewhat limited. Rather than employing high-performance external hardware, the focus is on intellectually overcoming the device's systematic bias through strategic experimental design. As part of this approach to mitigate device-to-device ADC variations and ensure grading fairness, three separate micro:bit units are provided, allowing students to systematically reduce potential disadvantages. Ultimately, this inquiry emphasizes the process of overcoming measurement limitations to achieve reliable results within a constrained environment.

Note: Significant figures are not reflected in the grading criteria for this problem.

Apparatus and Guidelines

1. Breadboard: A board used to build and connect your experimental circuits. A breadboard consists of power lines along both outer edges where holes are connected entirely in vertical columns to supply power (+) and ground (-), while the middle component area features internal connections in horizontal rows of exactly five holes each (A–E and F–J). These left and right five-hole groups are completely isolated from each other by the central divider running down the middle.

2. Precision Resistor Sets (A, B, C, and D): High-precision components with a 0.1% tolerance, where the known resistors serve as reference "true values" to verify system accuracy, while the unknown resistors are shielded with colored tubing for experimental determination—meaning any attempt to remove or damage these covers to uncover the hidden values will be strictly penalized as exam misconduct.

- A set of resistors A: ten $3.3\ \mathrm{k\Omega}$ resistors
- A set of resistors B: a $6.8\ \mathrm{k\Omega}$ resistor , a $560\ \Omega$ resistor, and a resistor of unknown value (covered by a green tube. Not used in this experiment).
- A set of resistors C : a $12\ \mathrm{k\Omega}$ resistor and two resistors of unknown value (one covered by a blue tube and the other by a red tube).
- A set of resistors D: a $510\ \mathrm{k\Omega}$ resistor and a resistor of unknown value (covered by a black tube).

Participants with color blindness, please raise your flag. A staff member will assist you.

3. Micro:bit Kit: A complete microcontroller setup consisting of a Micro:bit, an expansion board(3a), three pre-connected jumper wires(3b,3c,3d), and a battery holder(3e) equipped with AAA alkaline batteries.

Measuring Electric Potential and Instrumentation with provided Micro:bit kit

1. Wiring Guide (Refer to Fig. 1)

Connect the three jumper wires from the expansion board according to their vertical positions:

- 3b: Top Jumper Wire (top pin of expansion shield in Fig.1). Touch this to the specific node where you want to measure the voltage.
- 3c: Middle Jumper Wire (middle pin of expansion shield in Fig. 1). Connect to the circuit's power input (Vc,+) node.
- 3d: Bottom Jumper Wire (bottom pin of expansion shield in Fig. 1). Connect to the circuit's ground (GND, -) reference node.

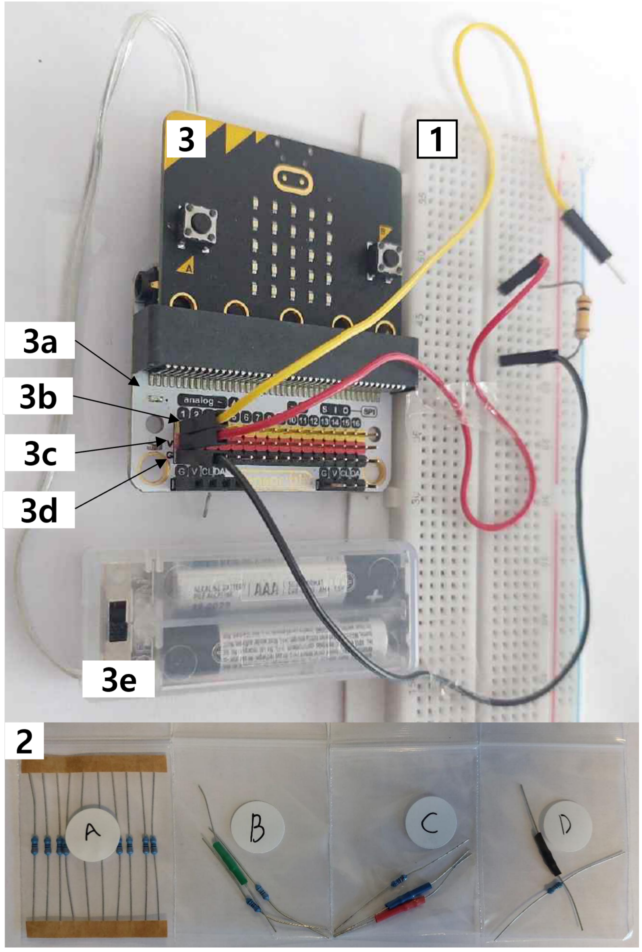

Fig.1

2. Measurement Procedure

    1. Power Setup: Connect the Bottom (GND) and Middle (Vc) wires first, then switch the battery pack ON.
    2. Pin Contact: While the power is on, touch the Top jumper wire to the target node.
    3. Data Entry: Press Button A or B on the front of the Micro:bit and record the scrolling digital value (N).

3. Important Notice

    - Switch off the battery pack when not in use to conserve power.
    - System Check: Connect the circuit as in Fig. 1. When you touch the Top wire to a point near the GND node, the reading should be near 0. When you touch the Top wire to the Middle wire's connection point (Vc), the reading should be near 1023. Verifying both points ensures that the measurement system's zero reference and full scale are correctly calibrated. If the measured values differ significantly from theoretical predictions, it is likely a wiring error or poor contact rather than a hardware limitation.
    - Prevent Shorts: Ensure the Middle (Vc) and Bottom (GND) wires do not touch each other directly without a load, as this can cause equipment failure. If a failure occurs, measurements will become

unavailable and an error will be indicated on the display; in such cases, a replacement device will be provided only once per participant.

- No Simultaneous Use: Connecting more than one Micro:bit to a single circuit may cause system failure and is strictly prohibited in this examination hall
- Linear Mapping: Excluding the discrete step effects arising from the quantization process, a linear proportional relationship exists between the node voltage and the integer measurement outcome N in an ideal ADC model. That is, potentials ranging from 0 V to Vc are linearly mapped to digital values between 0 and 1023 in an ideal ADC model.

Part A. Exploring the Meaning of ADC Measurement Values (1.2 pts)

The objective is to investigate the correlation between the Micro:bit's digital output (N) and the node potential (V). By constructing a network using **eight identical resistors** from 'Resistor Set A', you will examine whether the built-in ADC truly conforms to an ideal linear model based on the collected data. For the measurement process, the Micro:bit's pin is assumed to be an ideal probe that does not disturb the circuit's potential distribution. Assuming all resistors have identical values, this part aims to logically infer whether a proportional relationship between voltage and ADC readings actually holds, by cross-referencing the data with the circuit's structural characteristics without the use of external measuring devices.

A.1	Experimental Design and Circuit Schematics Design a resistor network that achieves the stated experimental objectives and provide a corresponding circuit schematic. When drawing the schematic, adhere to the following guidelines: Mark all nodes where potential measurements will be taken and assign a unique identifier to each (e.g., $a, b, ...$). Clearly label the connection points between the Micro:bit expansion board's power jumpers (V_c, GND) and the resistor network as V_c and GND, respectively.	0.6 pt

A.2	Data Collection and Analysis Conduct the experiment on the designed circuit using a single Micro:bit to obtain measurements for all nodes. Calculate the differences in ADC measurements between adjacent nodes to check linearity, and determine linearity based on a maximum deviation of 2% (single largest difference between any individual data point and the average value of the dataset).	0.6 pt

[Instructions for Subsequent Experiments]

Upon completion of Part A, retain only two of the resistors used in the experiment and return all others to their original container (Resistor Pack A). This is to ensure they are not mixed with other components in the following experimental procedures.

Part B. Simple Estimation and Error Analysis of Unknown Resistance Using a Known Reference (3.5 pts)

The objective of this part is to estimate the value of an unknown resistance r using a known reference resistor R and the Micro:bit's ADC output. The following assumptions are made for the analysis:

- ADC Linearity: The internal ADC is assumed to maintain a linear relationship between potential and the digital output N.

- Notation for Calibration Constants: N_L and N_H represent the readings at the GND and V_c nodes, respectively. (Ideally, $N_L = 0$ and $N_H = 1023$).
- Setup: Select any two resistors from Part A to serve as the reference (R) and the unknown (r).

B.1	Circuit Design and Schematics Design a measurement circuit to measure an unknown resistor (r), using a reference resistor (R) . Provide a corresponding circuit schematic. Note: The reference resistor R must be directly connected to the GND node. Label all measurement nodes with letters and specify all related variables in the circuit schematic. Clearly label the connection points between the Micro:bit expansion board's power jumpers (V_c, GND) and the resistor network as V_c and GND, respectively.	0.5 pt

B.2	Derivation of Estimation Formulas Using the voltage divider rule, derive an expression for the unknown resistance. Start by establishing the relationship between node potentials and derive the final formula using ADC output values in terms of N_L, N_H, and so on. Then rewrite the formula in the case of $N_L = 0$ and $N_H = 1023$.	0.5 pt

B.3	Experiment and Data Analysis Perform measurements using three Micro:bits, taking one reading per node for each device, and record the data. Assuming the true values of both the known and unknown resistance are $3.3\ \mathrm{k\Omega}$, calculate the mean relative error ($\bar{\epsilon}$) averaged over 3 ADCs and the relative standard deviation (RSD) of the estimated resistance values. Mean relative error is the average magnitude of the deviation between measured and true values, normalized by the true value. RSD is the ratio of the standard deviation to the mean.	1 pt

B.4	Analyze how the ADC measurement error (e) affects the accuracy of the estimated resistance. For simplicity, assume $N_L = 0$ and $N_H = 1023$. Derive a relationship for the resulting resistance error Δr in terms of N, e and R. Assume that e is sufficiently small compared to N.	0.6 pt

B.5	Resistance Range for a Given Error Tolerance Assuming an ADC error of $e = 1$, calculate the required range ofr/R for which the relative error of the resistance ($\frac{\Delta r}{r}$) remains within 1%.	0.9 pt

[Instructions for Subsequent Experiments]

Upon completion of Part B, return all to their original container (Resistor Pack A). This is to ensure they are not mixed with other components in the following experimental procedures.

Part C. Improving Resistance Estimation (1) :Overcoming The Problem of Quantization Error (1.9 pts)

This experiment aims to achieve more accurate resistance estimation under conditions where the basic method from Part B is insufficient due to quantization error. Quantization error is the difference between the continuous analog input signal and its discrete, digitized representation resulting from the finite resolution of an Analog-to-Digital Converter (ADC). When the resistance ratio (r/R) deviates significantly from 1, the quantization error (a single-step difference in ADC reading) is disproportionately amplified, leading to substantial errors in the estimated resistance. Resistor Set C is used in this part.

C.1	Initial Resistance Estimation and Statistical Analysis Using the reference resistor ($12\ \mathrm{k\Omega}$) from Resistor Pack C, estimate the values of unknown resistors r_1 (blue tube) and r_2 (red tube) respectively, based on your Part B method. Calculate the estimated resistance for one of the three Micro:bits.	0.6 pt

C.2	Experimental Design for Precision Enhancement Based on the results from C.1, the precision or reliability of the estimated values obtained for the unknown resistor r_2(red tube) may not be entirely satisfactory. This is due to the physical characteristic where estimation error follows a U-shaped curve depending on the resistance ratio (r/R). To address this issue and achieve a more accurate estimation of r_2, design an experimental scheme and provide the necessary circuit schematics with R, r_1, r_2. Note: All schematics must clearly label the measurement nodes and associated variables. Clearly label the connection points between the Micro:bit expansion board's power jumpers (V_c, GND) and the resistor network as V_c and GND, respectively.	0.6 pt

C.3	Experiment and Data Analysis Re-estimate the value of r_2 based on the new experimental design proposed in C.2. Calculate the resistance for each of the three Micro:bits and calculate the difference between the maximum and minimum values of the estimated resistance.	0.7 pt

[Instructions for Subsequent Experiments]

Upon completion of Part C, return all to their original container (Resistor Pack C). This is to ensure they are not mixed with other components in the following experimental procedures.

Part D. Improving Resistance Estimation (2) (3.4 pts)

Resistor Pack D contains a known resistor $R_1 (= 510\ \mathrm{k\Omega})$ and an unknown resistor r. Applying the simple estimation method from Part B may lead to significant measurement errors. The discrepancies observed in this task arise from a distinct physical factor—one that is fundamentally different from both the ADC nonlinearity and the amplification of quantization errors (Part C). Your challenge is to identify this underlying factor and develop a measurement strategy to achieve an accurate estimation of r.

D.1	Circuit Design and Schematics Devise a method to improve accuracy of estimating unknown resistor r, and provide a corresponding circuit schematic. Note: Label all measurement nodes and associated variables on the schematic. Clearly label the connection points between the Micro:bit expansion board's power jumpers (V_c, GND) and the resistor network as V_c and GND, respectively.	0.8 pt

D.2	Derivation of Estimation Formulas Based on your circuit design, derive the mathematical expressions to calculate the unknown resistance r. Start by establishing the relationship between node potentials and r, then derive the final formula using ADC output values ($N_H, N_L, , , ,$).	1.8 pt

D.3	Experiment and Data Analysis Implement your design and estimate the value of r. Calculate the estimated resistance for each of the three Micro:bits and derive the mean ($\bar{r}$) and calculate the difference between the maximum and minimum values of the estimated resistances.	0.8 pt