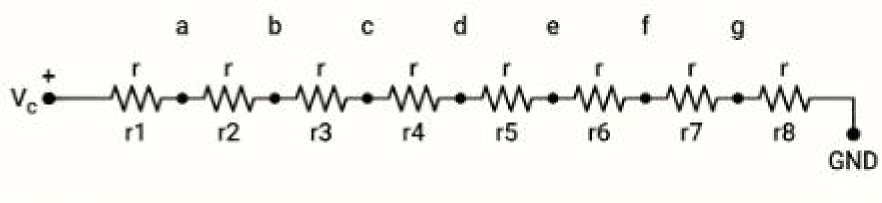
※ The reference value used to determine the accuracy of resistance estimation in this question is defined as twice the variance of the estimates obtained from 20 Micro:bit devices.
※ No marks will be awarded for the execution of the experiment if the experimental design fails to provide a valid solution to the problem.
※ No marks will be awarded if the student fails to present the requested numerical value in a form that the examiner can readily verify.

Part A

A.1)



Criteria	Points
If the circuit diagram connection (resistor ladder) is correctly expressed.	0.4
If all the measurement nodes are specified and correctly named	0.1
If V_c and GND nodes are correctly labeled	0.1

A.2)

Raw Data Table

	N_L	N_g	N_f	N_e	N_d	N_c	N_b	N_a	N_H
ADC1	0	128	257	385	513	639	769	897	1023

Derived Data Table

	ΔN (g/GND)	ΔN (f/g)	ΔN (e/f)	ΔN (d/e)	ΔN (c/d)	ΔN (b/c)	ΔN (a/b)	ΔN 126 (V_c/a)	$\overline{\Delta N}$
ADC1	128	129	128	128	126	130	128	126	127.88

$Max(\Delta N_i - \overline{\Delta N}) = 2.125$

maximum deviation =2.125/127.85*100=1.66%

☐ Linearity Verification (Within 2% Maximum Deviation)

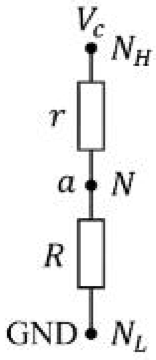
[√] Pass: The differences in ADC measurements between all adjacent nodes fall within the 2% threshold.

[ ] Fail: The deviation exceeds 2% at one or more nodes.

(Note: Depending on the specific Micro:bit unit used, a conclusion regarding linearity may or may not be drawn. The final conclusion itself is not critical; full credit will be awarded provided that the reasoning leading to the conclusion is rigorously justified.)

Criteria	Points
The raw data table includes columns for recording 9 measured node values.	0.1
The derived data table includes columns for the 8 differences between adjacent nodes	0.1
If the student's calculated value of average difference ($\overline{\Delta N}$), based on the data, is within a ±1% relative tolerance of the examiner's re-calculated value.	0.2
Points are awarded if the student's calculated maximum deviation directly matches the examiner's re-calculated value (derived from the student's raw data) within a ±10% relative tolerance.	0.1
If the student's conclusion regarding linearity matches the grader's recalculated data analysis	0.1

B-1)



Criteria	Points
If the circuit diagram connection (resistor ladder) is correctly expressed.	0.3
If all the measurement nodes are specified and correctly named	0.1
If V_c and GND nodes are correctly labeled	0.1

B-2)

To estimate the unknown resistance r, a known reference resistor R is connected in series with r. The voltage V at the junction, measured by the ADC, is determined by the voltage divider principle:

$$V = \frac{r}{R+r} V_c \quad (1)$$

Assuming a linear relationship between the input voltage V and the ADC output N, we define calibration constants N_L at GND and N_H at V_c

Due to linearity, voltage V can be expressed in terms of the digital reading N as:

$$V = \frac{N - N_L}{N_H - N_L} V_c \quad (2)$$

From (1), (2), unknown resister r satisfies:

$r = \frac{N_H - N}{N - N_L} R$ (3)

If $N_H = 1023$ and $N_L = 0$, then:

$r = \frac{1023 - N}{N} R$ (4)

Criteria	Points
If Equation (3) is correctly obtained (correctly expresses the unknown resistance as a function of R, N, N_L, N_H)	0.4
If Equation (1) is correctly obtained	0.2
If Equation (2) is correctly obtained	0.2
Correct result for Equation (4) when $N_L = 0, N_H = 1023$	0.1

B-3)

$r_{true} = 3.3K\Omega$

	N_L	N	N_H	$r_{est}\left(= \frac{N_H - N}{N - N_L} R\right)$	$\epsilon = \left\vert \frac{r_{true} - t_{est}}{r_{true}} \right\vert$
ADC1	0	512	1023	3294	,,,
ADC2	1	,,,	1022	,,,	,,,
ADC3	0	,,,	,,,	,,,	,,,

$\bar{\epsilon} =$

RSD=

※ Credit will be awarded only if the design in B.1 is correct.

Criteria	Points
If N, N_L, N_H for each device are all measured and recorded.	0.2
Points are awarded if the student's calculated value of unknown resistance directly matches the examiner's re-calculated value (derived from the student's raw data) within a ±1% relative tolerance. (Evaluate only the device for which	0.2

the student's estimate is most accurate among the three Micro:bit experiments.)	
If at least one of the unknown resistance values estimated by each Micro:bit falls within a 1% error range	0.2
Points are awarded if the student's calculated value of mean relative error directly matches the examiner's re-calculated value (derived from the student's raw data) within a ±1% relative tolerance. (The evaluation focuses solely on the accuracy of the process for calculating the relative error from the estimated resistance values, rather than the raw data.)	0.2
Points are awarded if the student's calculated value of RSD (Relative Standard Deviation) directly matches the examiner's re-calculated value (derived from the student's raw data) within a ±1% relative tolerance. (The evaluation focuses solely on the accuracy of the process for calculating the relative error from the estimated resistance values, rather than the raw data.)	0.2

B-4)

$$r = \frac{1023 - N}{N} R$$

$$\frac{dr}{dN} = -\frac{1023}{N^2} R \qquad (1)$$

$$e = \Delta N$$

$$\Delta r \approx \frac{dr}{dN} e = -\frac{1023}{N^2} Re \qquad (2)$$

Criteria	Points
If Equation (1) (dr/dN) is correctly	0.2

obtained	
If Equation (2) $(\varDelta r \approx \frac{dr}{dN}e)$ is correctly obtained	0.2
If Equation (2) for $\varDelta r$ is correctly obtained	0.6

B-5)

$$r = \frac{1023-N}{N}R$$

$$\varDelta r = -\frac{1023}{N^2}Re$$

ADC error $e=1$,

$$\left|\frac{\varDelta r}{r}\right| = \frac{1023}{N(1023-N)} \leq 0.01 \qquad (1)$$

$$N^2 - 1023N + 102300 \leq 0 \qquad (1*)$$

$$N \geq N_{\min} \approx 112.33 \text{ and } N \leq N_{\max} \approx 910.67 \qquad (2)$$

$r = \frac{1023-N}{N}R$에서

$$r_{\max} = \frac{1023-112.33}{112.33}R \approx 8.11R, \ r_{\min} = \frac{1023-910.67}{910.67}R \approx 0.123R \qquad (3*)$$

thus.

$$0.123 \leq r/R \leq 8.11 \qquad (3)$$

$$0.12 \leq r/R \leq 8.1 \quad (3**)$$

Criteria	Points
If Equation (1) or (1*) is correctly obtained	0.4
If Equation (2) is correctly obtained	0.2
If Equation (3), (3*), or (3**) is correctly obtained . Full marks if the student's calculated range, when rounded, matches the inequality (3**)	0.9

Part C

※ Full credit will be awarded to any student-designed experiment that successfully solves the given problem while meeting the required conditions.

※ Maintaining the reference resistance as R fails to resolve the quantization error;

therefore, it is not recognized as a valid design, and no marks will be awarded.
※ The resistors used in this experiment are Class B resistors with a 0.1% tolerance; R is specified as $12k\Omega$, r_1 as 1600Ω, and r_2 as 240Ω.

C.1)

$R = 12k\Omega$

r_1 case:

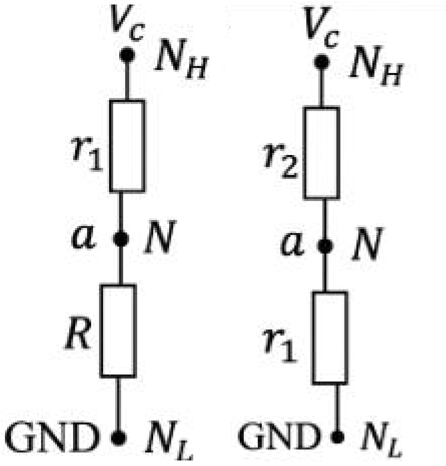
	N_L	N	N_H	$r_{est}\left(=\frac{N_H - N}{N - N_L}R\right)$
ADC1	0	904	1023	1580

r_2 case:

	N_L	N	N_H	$r_{est}\left(=\frac{N_H - N}{N - N_L}R\right)$
ADC1	0	1005	1023	215

Criteria	Points
N, N_H, N_L are recorded for each unknown resistor and device.	0.2 (0.1 per resistor)
Points are awarded if the student's calculated value of unknown resistance directly matches the examiner's re-calculated value (derived from the student's raw data) within a ±1% relative tolerance.	0.4 (0.2 per resistor)

C-2)



Note: Alternative approaches are also acceptable, such as estimating unknown resistor 1 using the reference resistor, and then using the parallel combination of the reference resistor and unknown resistor 1 as a new reference to estimate unknown resistor 2.
The method of estimating r_1 and r_1+r_2 using R and then subtracting them to find r_2 still suffers from quantization error issues, Thus is invalid design.

Criteria	Points
Circuit diagram is appropriate (demonstrates intent to use r_1 or an equivalent combined resistance of a similar size as a new reference resistance)	0.4
Measurement nodes are specified and correctly named.	0.1
V_c and GND nodes are appropriately labeled.	0.1

C.3)

$$r_1 = \frac{N_H - N}{N - N_L} R$$

$$r_2 = \frac{N_H - N}{N - N_L} r_1$$

	N_L	N	N_H	r_1	N_L	N	N_H	r_2
ADC1	0	904	1023	1580	0	891	1023	234

ADC2	0	904	1023	1580	0	891	1023	228
ADC3	0	906	1023	1550	0	892	1023	227

$\overline{r_2}$=

maximum-minimum (Δr_2) =

※ Credit will be awarded only if the design in C.2 is correct.

Criteria	Points
If the values of r_1 estimated in C1 are utilized for the estimation, and the measurements of N, N_H, N_L are recorded using the designated resistors as the reference resistor.	0.2
Points are awarded if the student's calculated value of unknown resistance r_2 directly matches the examiner's re-calculated value (derived from the student's raw data) within a ±1% relative tolerance. (Evaluate only the data from device for which the student's estimate is most accurate among the three Micro:bit experiments.)	0.2
If the calculated unknown resistance is accurately estimated within an range of $235 \pm 12\Omega$ using at least one device.	0.2
If the calculated Δr_2 (difference between maximum and minimum) from the student's data of resisters is correct	0.1

Part D.

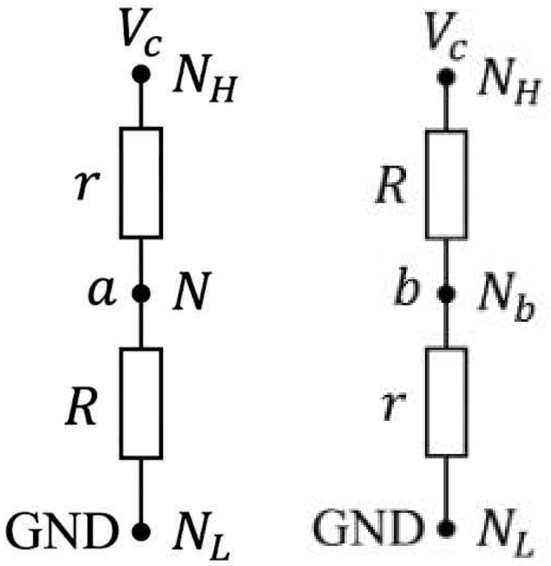
※ Full credit will be awarded to any student-designed experiment that successfully solves the given problem while meeting the required conditions.

※ "Zero marks will be awarded if multiple Micro:bits are used simultaneously within a single circuit, as this violation directly conflicts with the explicit prohibition stated in the question: “No Simultaneous Use: Connecting more than one Micro:bit to a single circuit may cause system failure and is strictly prohibited in this examination hall.

※ No marks will be awarded unless the **internal resistance issue** is explicitly resolved in a well-designed experiment and the resistance estimation formula derived from the experimental data is correct.

※ The resistors used in this experiment are Class B resistors with a 0.1% tolerance; R is specified as $510k\Omega$, r as $680k\Omega$.

D.1)



Criteria	Points
Circuit diagram includes a solution to estimate unknown resistance while considering internal resistance.	0.6
Measurement nodes are specified and correctly named.	0.1
V_c and GND nodes are appropriately labeled.	0.1

D.2)

Case 1: Unknown resistor (r) at the top, reference resistor (R) at the bottom. The voltage V_a is divided while R and the instrument's internal resistance R_i are in parallel.

Equivalent resistance at the bottom: $R_{eq1} = \frac{RR_i}{R+R_i}$

Voltage divider rule: $V_a = \frac{R_{eq1}}{R_{eq1}+r} V_c$

By combining thw two: $\frac{V_a}{V_c} = \frac{RR_i}{rR+rR_i+RR_i}$ (1)

Using the linear relationship between the ADC and voltage:

$\frac{N_a - N_L}{N_H - N_L} = \frac{V_a}{V_c}$ (2)

Case 2: Reference resistor (R) at the top, unknown resistor (r) at the bottom. Now r and R_i are in parallel.

Equivalent resistance at the bottom: $R_{eq1} = \frac{rR_i}{r+R_i}$

Voltage divider rule: $V_b = \frac{R_{eq2}}{R_{eq2}+R} V_c$

By combining thw two:

$\frac{V_b}{V_c} = \frac{rR_i}{rR+rR_i+RR_i}$ (3)

Using the linear relationship between the ADC and voltage:

$\frac{N_b - N_L}{N_H - N_L} = \frac{V_b}{V_c}$ (4)

From (1)~(4) the following is derived:

$r = \frac{N_b - N_L}{N_a - N_L} R$ (5)

Criteria	Points
If Eq. (5) is correctly obtained. (Full marks if the resistance estimation formula based on experimental values is correct, provided the experiment is	1.8

well-designed and explicitly addresses the internal resistance issue.)	
If Eq. (1) or (3) is correctly obtained (Points will be awarded if the voltage relationship formula taking internal resistance into account is derived)	0.5
If Eq. (2) or (4) is correctly obtained	0.3

D.3)

	N_L	N_a	N_b	N_H	r ($k\Omega$)
ADC1	0	450	589	1023	668
ADC2	0	450	591	1023	670
ADC3	0	449	589	1023	669

$\bar{r} =$

maxumum-minimum (Δr) =

※No credit will be given if the experiment is conducted using the basic method of PART B.

※ Credit will be awarded only if the design in D.1 is valid to overcome the issue of internal resistance.

Criteria	Points
If all measurements required to estimate the resistance for each device are recorded.	0.2
Points are awarded if the student's calculated unknown resistance is within a ±1% tolerance of the examiner's re-calculated value. (Only the data from the single Micro:bit experiment that yielded the highest accuracy among the three will be evaluated.)"	0.2
If at least one of the unknown resistance values estimated by each Micro:bit falls within the reference range, $668K\Omega \pm 5K\Omega$	0.3

Points are awarded if Δr values (difference between the maximum and minimum) is correctly calculated and verifiable from the student's data.	0.1